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Endurance of Multiplication Fact Fluency for Students With Attention Deficit Hyperactivity Disorder

Kelly K. Brady\(^1\) and Richard M. Kubina Jr.\(^1\)

Abstract
This study examines the relationship between a critical learning outcome of behavioral fluency and endurance, by comparing the effects of two practice procedures on multiplication facts two through nine. The first procedure, called whole time practice trial, consisted of an uninterrupted 1 minute practice time. The second procedure, endurance building practice trials, had three 20 second practice trials. A total of 3 students with attention deficit hyperactivity disorder participated. Results indicated that multiplication facts with the endurance building practice trials produced more efficient learning when compared to the whole time practice trial procedure for all 3 participants. Additionally, results show that even with the amount of practice time being equal, 1 minute in both conditions, on average participants practiced 30% more problems with the endurance building practice trials procedure than they did with the whole time practice trial procedure.

Keywords
endurance, behavioral fluency, precision teaching, Attention Deficit Hyperactivity Disorder

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Precision Teaching, a system that includes “defining instructional targets, monitoring daily performance, and organizing and presenting performance data in a uniform manner to facilitate timely and effective instructional decisions” (White, 2005, p. 1433), has a history in education that now spans over 40 years (e.g., Lindsley, 1964, 1968, 1971, 1990, 1997). Through the years and applications of Precision Teaching with many learners in schools and other educational settings, a concept called behavioral fluency came into existence. The theory of behavioral fluency posits that when a learner reaches a performance standard, a frequency range (e.g., typing 60-90 words per minute) three critical learning outcomes will also occur (Binder, 1996, 2005). The three learning outcomes are long-term retention, endurance or the ability to perform a behavior over a given period of time and with environmental distraction, and application or behavioral elements combining to form a behavioral compound.

A database has been steadily emerging that details the effects of attaining performance standards and the occurrence of the three critical learning outcomes associated with behavioral fluency (Kubina, 2010). Specifically, the research tentatively supports the proposition that behavioral fluency, as quantitatively defined as a performance standard, does produce skill retention and maintenance, skill endurance and subsequent resistance to distraction, and the application or combining of behaviors (Binder, 1996, 2005; Kubina, 2010; Kubina & Morrison, 2000). Although the bulk of behavioral fluency research exists for retention and application, there are fewer research studies focusing on endurance (Kubina, 2010). Endurance, or the ability to attend to a certain task for a given length of time and in the presence of environmental distractions (Binder 1984, 1996; Binder, Haughton, & Van Eyk, 1990), offers unique opportunities to understand and positively affect learning and behavior change programs.

For example, a study by Binder et al. (1990) examined the effects of endurance on writing fluency. A group of more than 75 students ranging from Kindergarten through eighth grade served as participants. The assignment had students writing digits from 0 to 9 as quickly as possible. As keeping all other conditions constant, teachers changed the length of task timings. Students wrote for 15 seconds, 30 seconds, 1 minute, 2 minutes, 4 minutes, 8 minutes, or 16 minutes intervals on different days. The results demonstrated that the students who reached a rate of 70 responses per minute performed at the same level regardless of whether the duration was 15 seconds or 16 minutes. Students who could not write quickly declined in performance as they progressed through the intervals. Some students, who wrote at about 20 digits per minute, stopped writing before the end of the 16 minute period. The study suggests that students
who have not attained minimal fluency levels, or lack endurance, cannot maintain their performance for any time period longer than brief durations before slowing down or stopping completely.

Based on the results of their study Binder et al. (1990) suggested that “attention span” may be thought of as endurance. In other words, before invoking complex cognitive processes to explain attention span, direct behavioral observations of endurance offer a parsimonious alternative. Another study showing the relationship between task fluency and increasing attention and endurance assessed a 9-year-old boy with attention deficit hyperactivity disorder (ADHD) on his level of endurance (McDowell & Keenan, 2001). The researchers assessed the student by measuring how many letter sounds were produced correctly and incorrectly per minute and how much time was spent on task. The results showed that as the participant became more fluent with the academic task, his attention and time spent on task also became more pronounced. By the end of the study, the student was performing at the fluency level for identifying letter sounds and was consistently maintaining his attention for 10 minute practice periods.

The studies on endurance have shown positive effects with attention and seem particularly well suited for students with ADHD. Endurance interventions directly target specific skills and attention. If endurance is a viable alternative for attention span then students with attention problems should greatly benefit from endurance interventions. In the present experiment we chose to examine endurance in applied setting and with a skill students needed for their current educational goals. The 3 participants, students with ADHD could not fluently multiply basic facts. To study endurance, we evaluated the difference in students’ performance in two equivalent conditions; a 1 minute multiplication practice trial and three 20 second practice trials. Both conditions were followed by a 1 minute assessment. As students with ADHD generally have difficulty paying attention for long time intervals we hypothesized that the students should learn more in a condition that maximizes their endurance as opposed to one that stretches it.

**Method**

**Participants and Setting**

There were three male students, Kurt who was in fourth grade and Miles and Mitch in fifth grade, who participated in the study. Each participant had a diagnosis of ADHD and received special education services as Other Health Impaired. All of the ADHD diagnoses were given by a physician or licensed
psychologist and were documented in the students educational files. Two of the students had comorbid conditions, Learning Disabilities. The study took place in a learning support classroom where students received instruction for reading, writing, and math as specified by their individualized education plans (IEPs). The classroom was located in a midsize northeastern university city where 14% of the students were classified as low income.

**Materials**

Materials used in the study included pretest screening probes. Each probe contained all of the basic multiplication facts 0-9. The probes were randomly generated using the single skill multiplication option from the Curriculum-Based Measurement Warehouse for Mathematics at http://Interventioncentral.com. Other materials were the multiplication probe sheets used during the intervention, pencils, and a countdown timer. The construction of the multiplication fact sheets is discussed in the following section.

**Dependent Variables**

The dependent variables were the number of correct and incorrect digits written per minute. Individual digits were defined as correct when the digit was in the proper position. For example, the answer 4 would count as one correct digit in response to the problem $2 \times 2 = \_$. Digits that were rotated or reversed, as long as the reversal or rotation did not affect the value of the number, were counted as correct digits. If the rotation or reversal did affect the value of the number by making it appear to be another number then it was counted as an incorrect digit. Individual digits were also defined as incorrect when the digit was the incorrect number to answer the problem. For instance, the answer 6 would count as one incorrect digit in response to the problem $2 \times 2 = \_$. If the digit was in an incorrect position or place value, for example answering 01 instead of 10 to the problem $2 \times 5 = \_\_$, the response would be counted as two incorrect digits. If only one digit was incorrect in a two digit answer, such as answering 23 to the problem $6 \times 4 = \_\_$, the response would count as one correct digit (the 2) and one incorrect digit (the 3). Skipped problems did not count for or against the total score. Sessions were conducted at approximately the same time each day for each participant in the same physical setting (i.e., their desk or a small table with the experimenter).

There were a total of $\times 8$ multiplication fact sheets. To control for difficulty, all problems containing a 0 or 1 were excluded during preassessment.
and during the construction of the 8 multiplication fact sheets. Math facts were generated to form a set of problems used with either the endurance building practice trials or the whole time practice trials on an alternating schedule. All problems from $2 \times 2$ to $9 \times 9$ were listed on a sheet of paper. Then, each problem along with its associate problem was assigned to one of the practice trial conditions. Associate problems were defined as a problem that would be found in the same fact family. For example, the associate problem of $6 \times 8$ would be $8 \times 6$. Problems with identical numbers such as $2 \times 2$ have no associate problems but these problems were distributed evenly by the alternating schedule.

Once all problems were assigned, the endurance building practice trials set contained 31 problems with the opportunity to write 58 correct digits per minute. The whole time practice trials set had 31 problems with the opportunity to write 60 correct digits per minute. Problems in both sets were of equal difficulty because of the split distribution used to create each set. Furthermore, an analysis of the students’ performance with each set before the intervention began showed that all students performed similarly with each set. Four sheets were created for each practice condition by taking the stock of problems for the set and randomizing all of the problems. The randomization step helped guard against the any serial memorization.

**Independent Variables**

The independent variables in this study were endurance building practice trials and whole time practice trials. Both conditions provided an equivalent amount of practice, 1 minute and then 30 seconds of corrective feedback. The conditions differed only in the breakdown and sequencing of the practice trials and corrective feedback. The endurance building practice trials condition allowed for three 20 second practice trials with immediate 10 second corrective feedback sessions following each practice trial. The whole time practice trial condition provided 1 minute uninterrupted practice trial followed by 30 seconds of corrective feedback. Therefore, both conditions had the same amount of practice time and the same amount of feedback with the only difference coming in the temporal segmentation of practice trials (i.e., three 20 second practice trials vs. one 60 second practice trial).

**Experimental Design**

An alternating treatments design (Cooper, Heron, & Heward, 2007) was used to examine the effects of the two practice conditions. Each participant was
evaluated using 1 minute assessment timing. During intervention, the two practice conditions were alternated daily over 4 weeks during the experimental sessions. Additionally, the practice conditions were counterbalanced to minimize multitreatment interference effects.

Procedure

Initial assessment. To determine current level of performance, each participant had to take five pretests containing math facts with numbers 0 through 9. They were timed for 1 minute and were told to stop and mark where they finished at the end of the minute. Then they were instructed to continue the rest of the problems that they knew on the sheet. The pretest determined when the students had acquired the requisite amount of fact to be considered in the instructional stage. For Grades 4 and up the instructional level for multiplication facts are 20-39 correct digits per minute (Deno & Mirkin, 1977). In the present study, two of the three students (i.e., Miles and Mitch) met the criteria for instructional level before beginning the intervention. Kurt did not attain instructional level before beginning the intervention but the experimenter decided to implement the intervention because it could still benefit him.

Intervention. Two different interventions, endurance building practice trials and a whole time practice trial, were introduced to the participants after the initial assessment. The endurance building practice trials procedure consisted of three 20 second practice trials interspersed with 10 second corrective feedback sessions, followed by a 1 minute multiplication math fact sheet assessment. The whole time practice trial incorporated a one 1 minute practice trial with a 30 second corrective feedback session, followed by a 1 minute multiplication math fact sheet assessment. The students sat at a desk across from the experimenter and were read standardized directions for each of the timings. Directions included the instruction to perform the skill as quickly as possible.

For each timing, the participants wrote the answers to the multiplication problems on each of the multiplication math fact sheets as quickly as they could for the amount of time allotted. Corrective feedback was given in each condition to correct errors and provide answers to skipped problems. During each of the practice trials procedure two timers were used. One timer was used to time the students’ performance on the multiplication math fact sheets and the other timer was set for either 10 seconds or 30 seconds to time the experimenter giving feedback. The two timers ensured equivalency of feedback between the conditions.

The error correction procedure used for both incorrect and skipped problems was a model-lead-check method (Stein, Kinder, Silbert, & Carnine, 2006).
For example, if the participant wrote the answer 5 to the problem $4 \times 4 = \_\_\_$, the examiner would restate the problem with the answer, ask the student to say the problem, and then ask the student to answer the problem. In both conditions, incorrect answers were corrected first and if time permitted the model-lead-check method was used with answers to skipped.

**Interobserver Agreement**

Earlier to baseline, an independent observer was instructed how to identify correct and incorrect digits for each probe. The independent observer was shown how to determine correct and incorrect digits, as described in the dependent variable section, and then asked to apply those rules and score a variety of hypothetical multiplication math fact sheets for correct and incorrect digits. This procedure was repeated until the experimenter and independent observer reached 80% agreement on the identification of correct and incorrect digits. Interobserver agreement checks were conducted for 100% of baseline sessions and 100% of intervention sessions. Using total agreement (dividing the smaller total by the larger total and multiplying by 100), interobserver agreement for this study was 100%.

**Procedural Integrity**

Fidelity of treatment was observed using a checklist of procedures during 25% of intervention sessions. An independent observer was given the checklist and observed the experimenter to determine her consistency in applying the methods outlines in the procedure section. The results from the independent observer indicated that the experimenter followed the steps with 100% accuracy.

**Results**

Figures 1-3 show the results for Kurt, Miles, and Mitch. Each graph has an alternating treatments design with solid dots showing the assessment of the endurance building practice trials or the three 20 second practice trials. The open circles represent the measurement of the whole time practice trial or the one 60 second trial. Figure 1 shows the minute assessment frequencies for Kurt. Kurt’s whole time practice trial responses ranged from 8 to 15 correct per minute. His performance in the endurance building practice trials ranged from 6 to 22. As the two interventions are implemented a separation emerges in the endurance building practice trial data set with only one
instance where the two conditions meet on the second session. Kurt’s data show the endurance building practice trials produce a higher level of learning when compared to the other condition. Although not as rapid, Kurt does show an upward trend of answering multiplication facts in the whole time practice trial condition.

Figure 2 shows the results for Miles. The range for the endurance building practice trials was 26 to 45 per minute whereas Miles performance in the whole time practice trial condition ranged from 18 to 36. In all instances, Miles performed at a higher level in the endurance building practice trials condition. There is a wide degree of separation between the two conditions. Similar to Kurt, although Miles performed at a lower level in the whole time practice trial condition he still showed an increase answering multiplication facts per minute.

The last graph, Figure 3, displays the results for Mitch. Mitch has a range of 28 to 32 correct answers per minute for the whole time practice trial condition and a range of 22 to 36 for the endurance building practice trials condition. Mitch begins by answering more problems in the whole time practice trial condition with a crossover to a higher performance in the endurance building condition during the second session. On the sixth session Mitch also has a performance higher than his performance in the endurance building.
Figure 2. Mitch’s correct per minute frequency during endurance building practice trials and whole time trial conditions

Figure 3. Mile’s correct per minute frequency during endurance building practice trials and whole time trial conditions
practice trials condition. Mitch shows a modest increase in the number of multiplication facts answered per minute during the whole time practice trial condition. For the majority of data points Mitch performs at a higher level during the endurance building practice trials condition.

Beyond the alternating treatments design performances, the number of problems practiced within the multiplication fact sheets were calculated for both conditions. On average, the students practiced 30% more problems with the endurance building practice trials condition than they did in the whole time practice trial condition. Specifically, Kurt practiced 127 problems during the endurance building practice trials condition and only 81 in the whole time practice trial condition. Similarly, Miles practiced 286 to 213 problems and Mitch practiced 289 to 192 problems in the endurance building and whole time trial conditions.

Discussion

Endurance refers the ability to attend to a certain task for a given length of time and in the presence of environmental distractions (Binder 1984, 1996, 2005; Binder et al., 1990). In this study we hypothesized that students with ADHD would benefit from an intervention that directly focused on building endurance. The results of this study indicate the endurance building practice trials did indeed produce greater learning. The three alternating treatments design shows a separation of data paths favoring the endurance building practice trials condition for all participants.

Each of the students responded similarly to the endurance practice trial conditions but had slight differences. For example, Kurt’s first session shows he performed better in the whole practice trial condition but by the third session he answered significantly more multiplication problems on the 1 minute multiplication fact sheet, 17 versus 11 per minute. At the end of the study Kurt maintained a differential of seven more digits written correctly per minute (i.e., 22 vs. 15) with the endurance practice trial conditions. Among the 3 participants, Kurt started off with the lowest frequencies for both conditions but still responded positively to both practice conditions by steadily answering more multiplication problems correctly.

Miles data, Figure 2, shows the clearest separation of data paths within the 3 participants. Miles first performance was higher during the endurance building practice trials from the beginning and never overlapped with the whole time practice condition thereby showing the clearest effect for an endurance building condition. Even though Miles did not practice the problems in the endurance practice trial condition for 1 minute, his performance
for 1 minute on the multiplication fact sheet rapidly increased and was clearly superior to the other condition. The differential between responding on the first trial was eight more digits written correctly per minute, 26 versus 18 digits written correctly per minute. The last session data show a very similar differential with nine more digits written correctly per minute, 45 versus 36 digits written correctly per minute. Developing endurance with the brief practice trials was demonstrated very clearly by Miles’ data.

Mitch’s performance also supports the use of shorter practice trials for building endurance though are not as clear as Miles. Mitch began the session with a higher data point for the whole trial condition but for the majority of the other sessions, aside from the sixth, Mitch performed better with the endurance building practice trials. Mitch’s performance along with the two other participants support the findings by McDowell and Keenan (2001); building endurance produces a higher frequency of responding when compared to conditions that tax endurance.

On a practical level, the students’ performance data points to an important teaching practice; namely, using procedures that enhance endurance. All students’ learned in the whole time trial condition but did so faster when the practice method more readily augmented endurance. The endurance building condition had students practice for a shorter period thereby facilitating attention or more accurately sharpening stimulus control. With the increased attention (i.e., endurance) the students also practiced more problems overall when compared to the whole trial condition. In contrast, the whole trial practice condition may have produced fatigue resulting in a decreased number of problems the students could attend to and subsequently practice. Similar to previous studies (i.e., Berens, Boyce, Berens, Doney, & Kenzer, 2003; Kim, Carr, & Templeton, 2001; McDowell & Keenan, 2001) teachers examining other academic behaviors may discover interventions directed specifically at addressing and enhancing endurance.

The results from the present study not only demonstrate endurance functioning in a capacity of attending to a task for a given length of time, but also suggest endurance may function as a sensitive indicator for behavioral persistence. For instance, Berens et al. (2003) found how well students performed for 3 minutes predicted how well students would perform for 5 minutes. McDowell and Keenan (2001) also noted the effects of 1 minute performance and its relationship to performing for 10 minutes. In the present study, the shorter timing intervals (i.e., 20 seconds) were also positively related to changes on the larger timing interval (i.e., 1 minute). Future studies may further validate the use of shorter timing intervals for reliably predicting performance with longer time intervals. Such information would help assessment and intervention plans.
One last implication of this study is the fact that all participants were diagnosed with ADHD. Children with ADHD have been described as having a difficult time sustaining attention to relevant stimuli and typically have impulsive responses to nonrelevant stimuli (Barkley, 1991). Additionally, the American Psychiatric Association (2000) has defined ADHD as a “persistent pattern of inattention and/or hyperactivity-impulsivity that is more frequently displayed and more severe than is typically observed in individuals at a comparable level of development” (p. 85). Attention, as a cognitive construct however, does not serve a behavioral analysis well because attention reifies attending behavior (Dube, MacDonald, Mansfield, Holcomb, & Ahearn, 2004).

Attending behavior and the concomitant concept of attention span or how much time a person can concentrate or attend to an activity, benefits from a directly observable measure such as endurance. Binder et al. (1990) describes the idea as follows:

Until students attain certain minimum levels of speed and accuracy . . . they typically lack the ability to maintain steady performance levels for extended periods of time. . . . when learners approach fluency . . . they become able to work steadily for significant durations. . . . Endurance, or attention span, thus follows or is a byproduct of behavioral fluency. (p. 25)

As demonstrated by Kurt, Miles, and Mitch, as their performance levels for the three 20 second intervals improved so too did their ability to perform over the longer time interval of 1 minute. Stated differently, the participants’ performance with writing answers to multiplication problems improved most under a condition of focused repetition (i.e., endurance building practice trials) and improved modestly during a condition of extended repetition (i.e., whole time practice trial condition). At best, the construct of attention in clinical definitions of ADHD does little more than provide a broadly descriptive pattern of behavior. At worse, attention may suggest interventions that target the actual problem indirectly and perhaps incorrectly. Endurance interventions offer a direct, measurable, and reliable alternative for addressing many learning problems.

**Limitations**

First, although the directions for each probe were to work across the page and not skip to known problems, participants did not always follow the directions despite redirecting prompts. Skipping problems could have affected the students’ rate at which they were learning and developing fluency. Students skipping problems and not attempting harder problems also
meant that students could have been responding to easiest and then slighter harder problems rather than attempting all problems in the domain of problems selected for each multiplication facts sheet. Second, this study was ended due to the academic year ending. Therefore, with extended time the students would have been able to reach a fluency criterion and have extended time with both conditions thereby more clearly establishing the effects of each condition. And third, this study mainly focused on the first part of the definition of endurance and did not assess students’ responses in the face of environmental distractions.

Conclusion

The findings from the study support the behavioral concept of endurance and its importance as an intervention. Both practice interventions helped the three students with ADHD to improve their frequency of writing correct answers to basic multiplication problems. The endurance building practice trials condition, three 20 second practice trials, produced more rapid learning and a greater number of problems practiced when compared to the whole time practice trial condition, an uninterrupted 1 minute practice time. The findings support endurance, an outcome of behavioral fluency, suggesting the ability to attend to a certain task for a given length of time and in the presence of environmental distractions can be experimentally tested and verified. Furthermore, endurance provides an alternative, behavioral description of attention span, and a possible means to help students with ADHD to improve multiplication skills.

Note

1. All data are available on Standard Celeration Charts. For copies please email the second author at rmk11@psu.edu.

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